[This question paper contains 4 printed pages.]

Your Roll No..............

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Sr. No. of Question Paper : 4881

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) – DSC**

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting two parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

5. Use of Calculator not allowed.

1. (a) Define a group G. If the square of every element in a group G is the identity then prove that G is Abelian.

   (b) Prove that the set R^* of nonzero real numbers is a group under multiplication.

   (c) Describe each symmetry in D_4.

   Prove that D_4 is a group.

2. (a) Define the order of an element in a group G. Find the order of each element in the group U(10).

   (b) Let G be an Abelian group and H, K are subgroups of G. Then prove that

   $HK = \{hk : h \in H, k \in K\}$ is a subgroup of G.

   (c) Let G be an Abelian group with identity e. Then prove that $H = \{x \in G : x^2 = e\}$ is a subgroup of G.

3. (a) Show that $\mathbb{Z}_8$ is a cyclic group under addition modulo 8. Find all its generators.

   (b) Consider a permutation $\alpha = (1\ 3\ 4\ 7)(2\ 9\ 8)(6\ 9)(4\ 1\ 6)$.

(i) Express α as a product of disjoint cycles.

(ii) Find α^{-1}

(iii) Find $o(\alpha)$

(iv) Determine if α is even or odd permutation.

(c) Define order of an element of a group. If α is an element of group G and $o(a) = 40$. Find $\langle a^{22} \rangle$, $\langle a^{12} \rangle$.

4. (a) Let H be a subgroup of G and $a, b \in G$. Prove that either $aH = bH$ or $aH \cap bH = \emptyset$.

(b) Let H be a subgroup of group G. Show that the collection of all the left cosets of H in G is a group. Name the group.

(c) Let Φ be a homomorphism from group G to G′. Define Ker Φ. Show that $\Phi(a) = \Phi(b)$ if and only if a Ker Φ = b Ker Φ, for a, b in G.

5. (a) Prove that the intersection of any collection of subrings of a ring R is a subring of R.

(b) Show that the set $Q\sqrt{2} = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a field.

(c) Define characteristics of a ring. Show that the characteristics of an integral domain is zero or prime.

6. (a) State ideal test. Prove that the set $3Z = \{3n \mid n \in Z\}$ is an ideal of Z.

(b) Show that $\varphi : Z_4 \to Z_{10}$ given by $\varphi(x) = 5x$ is a ring homomorphism.

(c) Let $\Phi : R \to S$ be a ring homomorphism and B an ideal of S. Show that $\Phi^{-1}(B) = \{r \in R \mid \Phi(r) \in B\}$ is an ideal of R.