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Sr. No. of Question Paper : 1441

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A. (Prog.)

Semester : II - DSC

Duration : 3 Hours

Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

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1441

1. (a) If  $x$  and  $y$  are vectors in  $\mathbb{R}^n$ , then prove that  $\|x + y\| \leq \|x\| + \|y\|$ .

Also verify it for the vectors  $x = [-1, 4, 2, 0, -3]$  and  $y = [2, 1, -4, -1, 0]$  in  $\mathbb{R}^5$ . (5.5+2)

- (b) Prove that for vectors  $x$  and  $y$  in  $\mathbb{R}^n$ ,

$$(i) \quad x \cdot y = \frac{1}{4} (\|x + y\|^2 - \|x - y\|^2)$$

$$(ii) \quad \text{If } (x + y) \cdot (x - y) = 0, \text{ then } \|x\| = \|y\|.$$

(4+3.5)

- (c) Solve the systems  $AX = B_1$  and  $AX = B_2$  simultaneously, where

$$A = \begin{bmatrix} 9 & 2 & 2 \\ 3 & 2 & 4 \\ 27 & 12 & 22 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -6 \\ 0 \\ 12 \end{bmatrix}, \quad \text{and} \quad B_2 = \begin{bmatrix} -12 \\ -3 \\ 8 \end{bmatrix}$$

(7.5)

2. (a) Find the reduced row echelon form of the following

matrix :

$$A = \begin{bmatrix} 2 & -5 & -20 \\ 0 & 2 & 7 \\ 1 & -5 & -19 \end{bmatrix} \quad (7.5)$$

(b) Express the vector  $x = [2, -1, 4]$  as a linear combination of vectors  $v_1 = [3, 6, 2]$  and  $v_2 = [2, 10, -4]$ , if possible. (7.5)

(c) Define the rank of a matrix and determine it for the following matrix :

$$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix} \quad (1.5+6)$$

3. (a) Check if the following matrix is diagonalizable or not :

$$\begin{bmatrix} 3 & 4 & 12 \\ 4 & -12 & 3 \\ 12 & 3 & -4 \end{bmatrix} \quad (7.5)$$

- (b) Show that the set of all polynomials  $P(x)$  forms a vector space under usual polynomial addition and scalar multiplication. (7.5)

- (c) Give an example of a finite dimensional vector space. Check if the following are a vector space or not :

- (i)  $\mathbb{R}^2$  with the addition  $[x, y] \oplus [w, z] = [x + w + 1, y + z - 1]$  and scalar multiplication  $a \otimes [x, y] = [ax + a - 1, ay - 2]$ .

(ii) set of all real valued functions  $f: \mathbb{R} \rightarrow \mathbb{R}$

such that  $f\left(\frac{1}{2}\right)=1$ , under usual function

addition and scalar multiplication.

(1.5+3+3)

4. (a) Define subspace of a vector space. Further show that intersection of two subspaces of a vector space  $V$  is a subspace of  $V$ . (1.5+6)

(b) Define a linearly independent set. Check if  $S = \{(1, -1, 0, 2), (0, -2, 1, 0), (2, 0, -1, 1)\}$  is linearly independent set in  $\mathbb{R}^4$  or not.

(1.5+6)

(c) Define an infinite dimensional and finite dimensional vector space.

Consider the set of all real polynomials denoted by  $P(x)$ , and the set of all real polynomials of degree at most  $n$  denoted by  $P_n(x)$ . Describe a basis of  $P(x)$  and  $P_n(x)$  and mention if these are finite dimensional or infinite dimensional.

(2+4+1.5)

5. (a) Show that the mapping  $L : M_{nn} \rightarrow M_{nn}$ , defined as  $L(A) = A + A^T$  is a linear operator, where  $M_{nn}$  is set of  $n \times n$  matrices and  $A^T$  denotes the transpose of the matrix  $A$ . Find the Kernel of  $L$ .

(3+4.5)

- (b) Let  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation defined as  $L\{[a, b]\} = [a - b, a, 2a + b]$ . Find the matrix of linear transformation  $A_{BC}$  of  $L$ , with respect to the basis  $B = \{[1, 2], [1, 0]\}$  and  $C = \{[1, 1, 0], [0, 1, 1], [1, 0, 1]\}$ .

(7.5)

(c) Let  $L: V \rightarrow W$ , be a linear transformation, then define  $\text{Ker}(L)$ ,  $\text{Range}(L)$ . Further show that  $\text{Ker}(L)$  is a subspace of  $V$  and  $\text{Range}(L)$  is a subspace of  $W$ .

(1.5+1.5+2.5+2)

6. (a) For the linear transformation  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined as

$$L \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Find  $\text{Ker}(L)$  and  $\text{Range}(L)$ . (4+3.5)

(b) Let  $L: V \rightarrow W$  be a one-to-one linear transformation. Show that if  $T$  is a linearly independent subset of  $V$ , then  $L(T)$  is a linearly independent subset of  $W$ .

(7.5)

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(c) For the linear transformation  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ , defined as :

$$L\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

Find  $L^{-1}$ , if it exists.

(7.5)