[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1421 F

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A. (Prog.)

Semester : II – DSC

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) If x and y are vectors in R^n, then prove that $|x \cdot y| \leq (\|x\|)(\|y\|)$. (7.5)

P.T.O.

(b) Prove that for vectors x and y in R^n,

(i) $||x + y||^2 = ||x||^2 + ||y||^2$ if and only if $x \cdot y = 0$.

(ii) $||x + y|| = ||x|| + ||y||$ if and only if $y = cx$, for some $c > 0$. (3.5+4)

(c) Describe geometric interpretation of addition of two vectors in R^2. In particular, represent the sum of the vectors a = [3,4] and b = [−2,1] geometrically. (7.5)

2. (a) Find the reduced row echelon form B of the following matrix A :

$$A = \begin{bmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{bmatrix}$$

Also give a sequence of row operations that converts B back to A. (5.5+2)

(b) Determine whether the vector [5, 17, −20] is in the row space of matrix $\begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$. (7.5)

(c) Use the Gaussian elimination method to find the complete solution set for the following homogeneous system.

$$-2x + y + 8z = 0$$

$$7x - 2y - 22z = 0$$

$$3x - y - 10z = 0$$

(7.5)

3. (a) Find the eigen values and corresponding eigen vectors of the following matrix :

$$\begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & 8 & 14 \end{bmatrix}$$

(3+4.5)

(b) Define a vector space. Let V be a vector space, then for every vector v in V and every real number a, prove that

(i) a. 0 = 0.

(ii) if a. v = 0 then a = 0 or v = 0.

(1.5+3+3)

(c) Show that the set M_{nm} of all real matrices of order n × m forms a vector space under the matrix addition and scalar multiplication. (7.5)

4. (a) Prove or disprove, if the following sets are subspace of R^4:

(i) W1 = {(a, b, 0, 0): a, b ∈ R)

(ii) W2 = {(a, b, a – 1, 0): a, b ∈ R)

(4+3.5)

(b) Define span of a set S. Let S be a non-empty subset of a vector space V, then prove the following :

(i) $S \subseteq$ span (5).

(ii) span(S) is a subspace of V.

(1.5+2.5+3.5)

(c) Define basis of a vector space. Give an example of a basis for the following vector spaces :

(i) $P_n(R)$- set of all polynomials of degree at most n.

(ii) R^n- set of all n tuples of real numbers.

(iii) M_{nm}- set of all n * m real matrices.

(1.5+2+2+2)

5. (a) Define a linear transformation. If $L : V \to W$ is a linear transformation, and 0_v is the zero vector in V and 0_W is the zero vector in W, then prove the following :

(i) $L(0_V) = 0_W$.

(ii) $L(-v) = -L(v)$ for all $v \in V$.

(1.5+3+3)

(b) Let $L: P_3(x) \to R^3$ be a linear transformation. Find the matrix of linear transformation A_{BC} of L, with respect to the standard order basis $B = \{1, x, x^2, x^3\}$ of $P_3(x)$ and $C = \{e_1, e_2, e_3\}$ of R^3, where L is defined as :

$$L(dx^3 + cx^2 + bx + a) = [a + b, 2c, d - a].$$

(7.5)

(c) State Dimension theorem. Further prove it for the linear transformation

$L: P_3(x) \to P_2(x)$, defined as $L(ax^3 + bx^2 + cx + d) = 3ax^2 + 2bx + c$. (2+5.5)

6. (a) For the linear transformation $L: R^3 \to R^3$ defined as

$$L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Find Ker(L) and Range(L). (4+3.5)

(b) Define a one-to-one linear transformation. Show that a linear transformation $L : V \to W$ is one-to-one if and only if $Ker(L) = \{0\}$. (2+5.5)

(c) For the linear transformation L: $R^3 \rightarrow R^3$ defined as, L(v) = A. v, where

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 1 & 1 & -1 \end{pmatrix}$$

Determine, whether L is an isomorphism or not. (7.5)